Urban transportation

* Transport economics: Partial equilibrium, choice of commuting options.

* Urban way: Commuting mode choice from the urban economics perspective.
→ Full eq; but not as well suited for the data.

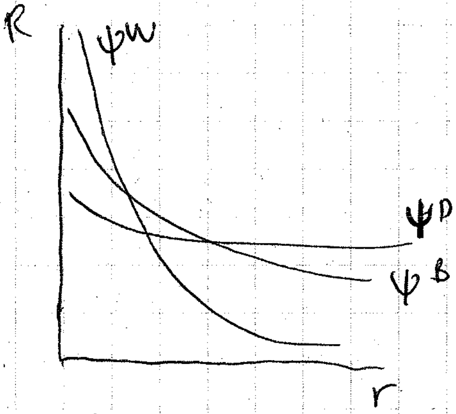
Commuting options:

	walk	$w(1-b_W r) = z + sR(r)$
	bus	$w(1-X-b_B r) = z + sR(r)$
Time cost important for high w ←	drive	$w(1-b_D r) - c = z + sR(r)$

Pecuniary cost important for low w.

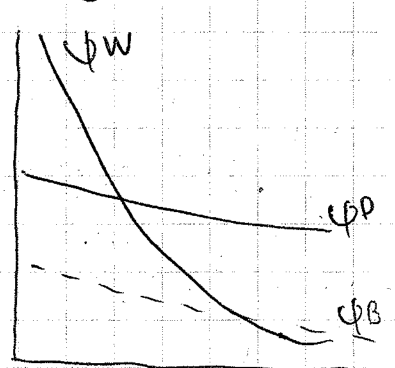
$b_W > b_B > b_D$ Time to travel 1 unit of distance.
X → Fixed time cost for bus
c → Fixed pecuniary cost of driving.

Slopes of bid-rents $-\frac{wb_W}{s}$, $-\frac{wb_B}{s}$, $-\frac{wb_D}{s}$

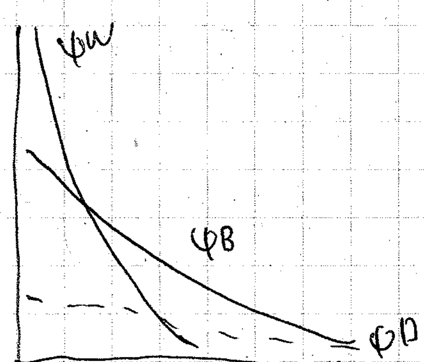


→ Seems to match configuration of big cities...

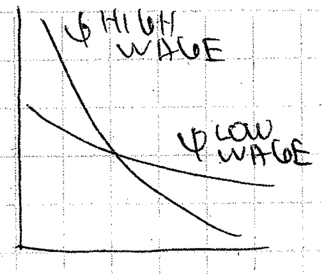
High wage



Low wage



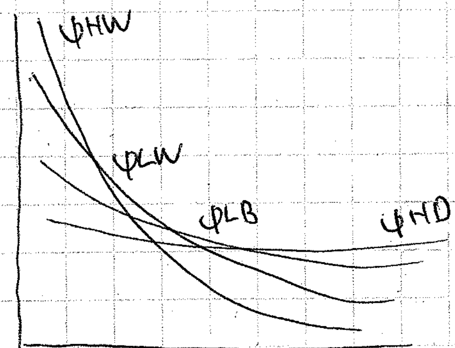
* Why do the poor live in cities? should be outbid by the rich.
→ Glaeser paper ⚠



if $\frac{d\ln s}{d\ln I} < 1$

→ should hold within each commuting mode.

Combining the two



→ shows forces at play, but does not suit data well

Modelling individual commuting mode choice

- McFadden (1974)

Random utility set-up

$$U(s, x) = V(s, f(x)) + \varepsilon(s, x) \quad \rightsquigarrow \text{indirect utility}$$

s: Indiv chars; x: choice chars; V: deterministic; ε: iid draw

$$Pr[x_i | s, B] = \prod_{i \neq j}^{I} Pr[U(s, x_i) > U(s, x_j)]$$

B: choice set

$\rightsquigarrow$ prob of utility greater than other choices

$$= Pr[U(s, x_i) \geq \max_j U(s, x_j)]$$

Need to choose distribution with closed form for max.

~~F(ε < y)~~ $F(y) = P(\varepsilon < y) = e^{-e^{-\left(\frac{x-\mu}{b}\right)}}$ Extreme value type I (Gumbel)

mode $= \mu$ median $= \mu - \ln(\ln(2))$

mean $= \mu + 0.58$

sd $= \pi / \sqrt{6}$

$\} \; b = 1.$

$$Pr[\text{choose } 1] = Pr[v + \varepsilon_1 \geq \max_{j>1} [v_j + \varepsilon_j]]$$

Distribution of max for EV type I: $\rightsquigarrow Pr[\varepsilon_2 \leq u - v_2] \, Pr[\varepsilon_3 \leq u - v_3] \ldots$

$$Pr[\max_{j>1} [v_j + \varepsilon_j] \leq u] = \prod_{j=2}^{I} e^{-e^{-(u - v_j)}}$$

$= e^{-\sum_j e^{-(u-v_j)}} = e^{-e^{-u} \sum_j e^{v_j}}$

$Pr[\max_{j>1} [v_j + \varepsilon_j] \le u] = e^{-e^{-u} \sum_{j>1} v_j}$

So $Pr[v_1 + \varepsilon_1 \ge \max_{j>1} \{v_j + \varepsilon_j\}] = e^{-e^{-(v_1+\varepsilon_1)} \sum_{j>1} v_j}$

Unconditional probability of choosing 1

$$= \int_{-\infty}^{\infty} f(\varepsilon_1) Pr(v_1 + \varepsilon_1 \ge \max_{j>1} v_j + \varepsilon_j)\, d\varepsilon_1$$

$$= \frac{e^{-(e^{v_1}(e^{v_2} + e^{v_3} + \cdots + e^{v_K}) + 1)} e^{-\varepsilon_1}}{e^{v_1}(e^{v_2} + e^{v_3} + \cdots + e^{v_K}) + 1} = \frac{e^{v_1}}{\sum_{j=1} e^{v_j}}$$

& switch in notation

$$P_{ij} = \frac{\exp(\phi_i^T s_i + \theta^T c_{ij} s_j)}{\sum_k \exp(\phi_k^T s_j + \theta^T c_{kj} s_j)}$$

choice chars

Estimate by ML

$\mathcal{L}(\phi, \theta) = \prod_j \left[\prod_i [P_{ij}]^{D_{ij}} \right]$ $= 1$ if j chooses i, 0 o/w

$j \to$ "What units"? Households, individuals? $\to$ Intrahousehold allocation Δ

$i \to$ IIA, criticism...

IIA may not hold given the information we have about choices.

Identification: Not all ϕ, θ identified, only can identify them relative to a reference choice.

$$P_{ij} = \frac{\exp[(\phi_i - \phi_1)^T s_i + \theta^T (c_{ij} - c_{1j}) s_j)}{1 + \sum_{k>1} \exp(\phi_k - \phi_1) s_i + \theta^T (c_{kj} - c_{1j}) s_j}$$

What to do to get at IIA?

$\to$ Normal disturbances

Probit random coefficients model

Probit random coefficients model

* Correlation in errors and coefficients across choices.

Suppose $U_i = z_i\beta + \eta_i$ for choice i → can be relaxed.
$\eta_i = N(0, \sigma^2_i)$, $\eta_i \perp\!\!\!\perp z_i, \eta_j, \beta$

$\beta \sim N(\bar{\beta}, \Sigma_\beta)$

$U_i = z_i\bar{\beta} + z_i(\beta - \bar{\beta}) + \eta_i$
Suppose there are three choices
If $U_1 \geq U_2$
$U_1 - U_2 = (z_1 - z_2)\bar{\beta} + (z_i - z_2)(\beta - \bar{\beta}) + \eta_1 - \eta_2 \geq 0$

$Var(U_1 - U_2) = (z_1 - z_2)\Sigma_\beta(z_i - z_2)^T + \sigma_1^2 + \sigma_2^2$
$Cov(U_1 - U_2, U_1 - U_3) = (z_i - z_2)\Sigma_\beta(z_i - z_3)^T + \sigma_1^2$

$$\rho = corr(U_1 - U_2, U_1 - U_3) = \frac{cov(U_1 - U_2, U_1 - U_3)}{\sqrt{V(U_1 - U_2) + V(U_1 - U_3)}}$$
→ specify correlation later.

Want to know $Pr[1 \mid \{1,2,3\}]$
$= Pr[U_1 - U_2 \geq 0$ and $U_1 - U_3 \geq 0]$
$= Pr[(z_i - z_2)\bar{\beta} + \sqrt{V(U_1 - U_2)}\,\epsilon_1 \geq 0$
and $(z_1 - z_3)\bar{\beta} + \sqrt{V(U_1 - U_3)}\,\epsilon_2 \geq 0]$ ϵ_1, ϵ_2 joint standard normal.

$Pr(\text{choose } 1) = \int\int f(\epsilon_1, \epsilon_2)\,d\epsilon_2\,d\epsilon_1$
↳ joint std normal

Limits
$$= \int_{-\infty}^{\frac{(z_1 - z_2)\bar{\beta}}{\sqrt{V(U_1 - U_2)}}} \int_{-\infty}^{\frac{(z_1 - z_3)\bar{\beta}}{\sqrt{V(U_1 - U_3)}}} f(\epsilon_1, \epsilon_2)\,d\epsilon_2\,d\epsilon_1$$

$$\mathcal{L}(\bar{\beta}, \Sigma_\beta, \sigma_1^2, \sigma_2^2, \sigma_3^2) = \prod_j \prod_i [[Pr \text{ choose } i]_j]^{D_{ij}}$$

Small / Winston / Yan

"Revealed preference" — F or X, cost on X, relative time, time of day → relative 80-50 pctile in time.

Not enough variation.

"Stated preference"

Variables: c_{it} toll difference
R_{it} unrelabiality difference
T_{it} time difference

Values of travel time $\frac{\partial V/\partial T}{\partial V/\partial C} = \frac{\partial V/\partial T}{\lambda} = \frac{\text{utils}/\text{time}}{\text{utils}/\$} = \$/\text{time}$

$U_1 = v_1 + \varepsilon_1$
$U_2 = v_2 + \varepsilon_2$

$Pr(1 \mid \{1,2\}) = \frac{e^{v_1}}{e^{v_1} + e^{v_2}} = \frac{e^{v_1 - v_2}}{1 + e^{v_1 - v_2}}$

$Pr(2 \mid \{1,2\}) = \frac{1}{1 + e^{v_1 - v_2}}$

$v_1 - v_2 = u_i - n_i$

$Pr(U_1 > U_2) = Pr$

$n_i \sim EV$
$\Delta n_i \to$ Logit.

$U_{it} = \theta_i + \beta_i \underbrace{X_{it}}_{\text{Choice chars}}$

$VOT_i = \frac{\partial U_{it}/\partial T_{it}}{\partial U_{it}/\partial C_{it}}$ $\quad VOR_i = \frac{\partial U_{it}/\partial R_{it}}{\partial U_{it}/\partial C_{it}}$

Heterogeneity! $\theta_i = \bar{\theta} + \phi w_i + \xi_i \sim N(0, \sigma^2_\xi)$
$\beta_i = \bar{\beta} + r z_i + \xi_{\beta i} \sim N(0, \Omega)$

Some more assumptions on error structure, final system

Estimation.

xpress freeway

Example for two choices: logit $\quad 1 = X \quad 2 = F$

$$U_1 = X_{1i}\beta + n_1 \rightarrow P_r[X] = P_r[(X_{1i} - X_{2i})\beta > n_2 - n_1)$$
$$U_2 = X_{2i}\beta + n_2 \qquad = \frac{e^{(X_1 - X_2)\beta}}{1 + e^{(X_1 - X_2)\beta}}$$

$\rightarrow$ 1 if X

$$\mathcal{L} = \prod_j \left[\frac{e^{(X_{1j} - X_{2j})\beta}}{1 + e^{(X_{1j} - X_{2j})\beta}}\right]^{D_j} \left[\frac{1}{1 + e^{(X_{1j} - X_{2j})\beta}}\right]^{1 - D_j}$$

Their model

$$\mathcal{L} = \prod_j \int \prod_t \underbrace{P_r(X \mid \phi, \theta)^{D_{jt}} P_r(F \mid \phi, \theta)^{1 - D_{jt}}} f(\theta)\, d\theta$$

Individuals (j); Obs within individual (t)

$\rightarrow$ Choices independent conditional on the random effects from θ.

$$P_r(X \mid \phi, \theta) = F[\theta^{BK} + \phi^{BK} w_i + \beta_i^{BK} X_i^{BK} + v_i^{BK}]$$
or
$$F[{}^{BS}] \ldots$$

c does not enter integral, only one obs, not correlated.

Algorithm

1. θ^0
2. simulate L, get θ^1 by gradient based method
3. Stop when $|\theta^{k+1} - \theta^k| <$ stopping crit.

Policy paper $\rightarrow$ HOV simulations