Chapter 6
Spatial Equilibrium and Systems of Cities

In the previous chapter, we had a first look at the spatial distribution of economic activity within a country or region. We asked where the factors of production would locate, especially looking at the role of transport costs and trade. We now turn to a related question: what determines the spatial variation in factor prices? Why are wages and rents different in different locations, and what can we learn from their distribution in space?[1]

We will look at two papers in this section: First, we will introduce the concept of spatial equilibrium in Roback (1982). There we will learn what happens when capital and labor are free to move across locations, and how that helps us understand the role of local characteristics.

Second, we will look at Diamond (2016). This paper builds and estimates a quantitative model of the US system of cities. She uses it to study the relative strength of a number of economic forces in shaping the current spatial distributions of wage, labor, and amenities in the US. The paper is technically complex, but we will focus on the research question, the need for a structural model, what in the data identifies the parameters she cares about, and results.

6.1 Wages, Rents and the Quality of Life

We will lay out a spatial general equilibrium model. Despite being very simple, it will yield a lot of intuition about the role of space when factors are mobile. Cities differ by their amenities and productivity. Wages and rents will be determined by the model.

Let s denote the level of an amenity. Cities differ by their level of s. We can think of s as a local characteristic that affects how pleasant it is to live in a location - say frequency of sunny days or air quality. The amenity level s can also affect production: say if clean local air comes at the expense of strict regulation. For the

[1] This chapter was prepared by Lorenzo Aldeco

rest of this section we will assume that larger s implies larger production costs for firms, but the model can accommodate the two other cases as well.

Residents of each city produce and consume a good, X, that is traded costlessly across cities and whose price is given and normalized to 1. Workers and firms are perfectly mobile, and choose where to locate to maximize utility and profits. There is a fixed amount of land in each city, but it can be used for housing or production.

Workers are all identical in skills and tastes. Each worker supplies a single unit of labor inelastically, wherever she locates.

A worker living in a location with amenity level s solves the following problem. x is good consumption, l^c is land consumption, w is the local wage, and r is rent.

$$\max U(x, l^c; s) \tag{6.1}$$

$$\text{s.t. } w = x + l^c r \tag{6.2}$$

In a spatial equilibrium, prices and quantities in each location are such that no person wants to move and markets clear. Then, the value function V must be such that, for all possible amenity levels, wages and rents adjust to equalize utility.

$$V(w, r; s) = k \tag{6.3}$$

Firms produce using labor and land using technology with constant returns to scale. Define l^p to be land used in production.

Production is given by:

$$X = f(l^p, N; s)$$

For no firms to move, unit cost C must equal price in each location:

$$C(w, r; s) = 1 \tag{6.4}$$

If any location had price larger than unit cost, all firms would want to locate there. This on its own is not a problem, but if all production happened in the same place, all labor would live in the same place too. Since land is scarce, rent would be much higher there than in an unoccupied location with marginally different productivity. Then, firms and workers could get a discrete decrease in rent at infinitesimal cost by moving.

Notice, for future reference, that if wage increases, unit cost increases by the labor cost per unit. (The same applies to increases in rent).

$$C_w = N/X$$
$$C_r = l^p/X$$

6.1.1 Equilibrium

First, we will look at comparative statics. What happens in equilibrium when a city increases its level of amenities?

Intuitively, in each location, indexed by s, wages and rents must be such that utility equals k and unit cost equals 1.

Since we know how amenities affect utility and cost, we can draw the isoquants corresponding to the equilibrium conditions (6.3) and (6.4)

Given s, the point where they intersect will correspond to the equilibrium wage and rent, as seen in Fig 6.1

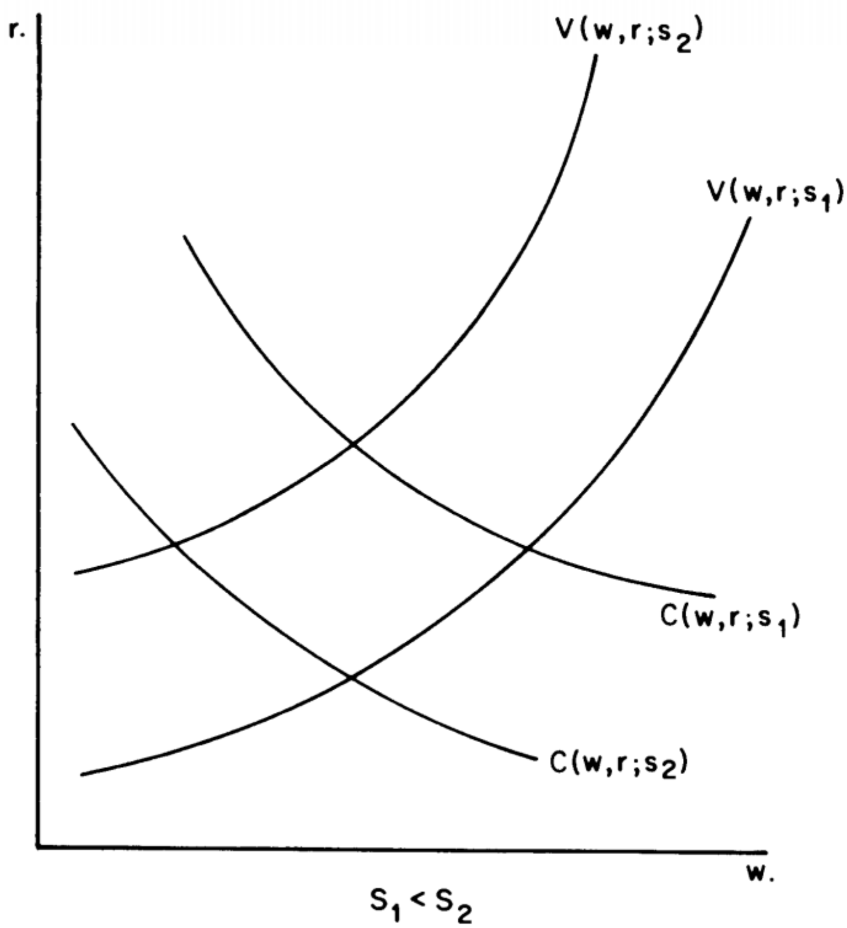

Fig. 6.1 Comparative statics: amenity change

Then, if local amenities change from s_1 to s_2, both curves will shift. Consider the isocost curve. First, note it slopes down. For larger rent, wages must decrease to keep costs constant. For fixed wage if s increases, since the amenity is unproductive, rent must decrease to keep cost constant. Then, the curve shifts inward.

The iso-utility curve, on the other hand, slopes up. If wage increases, rent must also increase to keep utility constant. For fixed wage, when s increases, since it improves utility, rent must increase to keep utility constant. Then, the curve shifts up.

Then, in places with high amenities, wages will be unequivocally lower. Workers accept lower wages in order to live with high s. Firms, on the other hand, must be compensated with lower costs in order to produce under high s.

The effect on rent will depend on whether s improves utility more or less than it harms production. In order to see this, we need to dive into the math. Fortunately, the results are very general and we won't need to explicitly solve the model to reach them.

We want to understand two things:

- the effect of amenities on consumer welfare
- the effect of amenities on firms' costs

Start by differentiating (6.3) and (6.4) w.r.t s.

This lets us express $\frac{\delta w}{\delta s}$ and $\frac{\delta r}{\delta s}$ as follows:

$$\frac{\delta w}{\delta s} = \frac{C_s V_r - C_r V_s}{C_r V_w - C_w V_r} \quad < 0 \tag{6.5}$$

$$\frac{\delta r}{\delta s} = \frac{C_w V_s - C_s V_w}{C_r V_w - C_w V_r} \quad \gtreqless 0 \tag{6.6}$$

Note the denominator is the same in both expressions. We can simplify it. Recall the consumer's problem (6.1). From its Lagrangian, with multiplier λ it follows that

$$\begin{aligned} \frac{\delta V}{\delta w} &= \frac{\delta U(x^*, l^{c*}; s)}{\delta w} \\ &= \lambda \\ \frac{\delta V}{\delta r} &= \frac{\delta U(x^*, l^*; s)}{\delta r} \\ &= -\lambda l^{c*} \end{aligned}$$

Where the first and third equalities follow from the Envelope Theorem.

Then, the ratio of marginal indirect utilities yield the Marshallian demand (this is Roy's identity):

$$\frac{V_r}{V_w} = -l^{c*}$$

Factorizing V_w from the denominator in (6.5), and substituting in the derivatives of unit cost yields

$$C_r V_w - C_w V_r = \frac{L(s)V_w}{X}$$

Equations (6.5) contain the two objects we are after. C_s is evidently the effect of amenities on cost. How can we know the effect of amenity on consumer utility? In principle V_s tells us just this, but V_s is expressed in utils per amenity unit, and so is not helpful. However, V_w is the marginal effect of income on utility, so

$$p_s^* = \frac{V_s}{V_w}$$

is the willingness to pay for a marginal amenity increase, in dollars per amenity unit.

Solving equations (6.5) for $\frac{V_s}{V_w}$ and C_s yields:

$$p_s^* = l^c \frac{\delta r}{\delta s} - \frac{\delta w}{\delta s} \tag{6.7}$$

$$C_s = -\left(\frac{\delta w}{\delta s}\frac{N}{X} + \frac{\delta r}{\delta s}\frac{l^p}{X} \right) \tag{6.8}$$

All this says is that consumers pay for more amenities through increased land costs and decreased wages, and that firms are compensated by lower labor and land costs.

We can now calculate the total effect of a marginal amenity increase on welfare, starting from level s_0 , as:

$$p_s^* N(s_0) - C_s X(s_0) = \frac{\delta r}{\delta s} L(s_0)$$

This is a striking result. The RHS of the above equation is the change in land value: change in price times total quantity. This result then states that the welfare effect of an increase in amenities is exactly equal to the change in land values, when factors are mobile. Once we make sense of this, we acquire a new intuition about how space matters. When factors are mobile (that is, in spatial equilibrium), any local differences must be reflected in the price of the only immobile factor: land.

It is partly for this reason that land prices are so important and useful for empirical applications. High rents in a location imply there is something of value, either for consumption or production, at that location.

6.1.2 Empirical Applications

There are a number of ways we can bring the model to the data. These all revolve around estimating $\frac{\delta w}{\delta s}$ and $\frac{\delta r}{\delta s}$, by regressing wages on a set of observable amenities.

First, we can calculate the implicit prices of different local characteristics, using (6.7). This is one of Roback's main objectives. She calculates the dollar price of several local characteristics, including crime, pollution, and weather.

Second, we can ask how much of the variation in wages across space is explained by differences in local amenities. We can know this by looking at the R^2 of a regression of wages and local amenities - this exercise shows that half the variation in wages being explained by a few important amenities. It also shows that amenities explain most of the interregional variation.

The model can be partially tested. If it is correct, things that we think of as disamenities should be correlated with higher wages, and amenities with lower ones. We can check this in the data - and in Roback's setting the observed relationships are indeed consistent with the model.

While it is not clear in this setting, there is another economic quantity that signals there is something valuable at a given location: the amount of factors of production. More attractive and productive places will attract larger amounts of mobile labor and capital in equilibrium, even if prices push to equalize utility and profits. This will become clearer when we look at the next paper.

6.2 The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000

This paper starts out by noting that from the 1980's on, the gap between skilled and unskilled wages has consistently gotten larger. At the same time, skilled workers have concentrated in space: cities with high shares of skilled labor in 1980 attracted even more skilled workers in the years since. This geographic concentration is economically important.

Local increases in rents might mitigate the welfare differences between skilled and unskilled - and we do observe rents rising in high skill cities. On the other hand, amenities also play a role: if culture, good quality schools, and infrastructure follow higher income workers, then the gap in welfare might be larger. Finally, agglomeration will further modify local wages, complicating matters.

Diamond's paper wants to answer:

1. How much of the geographic sorting by skill is due to local demand for skilled labor, and how much due to endogenous changes in amenities, agglomeration, and rents?
2. How important is that amenities respond to the share of skilled workers in a city? Does it explain a lot of the observed distribution of skilled workers?
3. Has the welfare gap between skilled and unskilled workers increased as much as suggested by the increase in the wage gap?

All these are quantitative questions, but they cannot be answered by estimating a reduced form equation. They are essentially questions about counterfactuals. To

Fig. 6.2 Wage changes in metropolitan areas, by skill, from Moretti (2011)

make this clear, we can restate each question, assuming we have a working model of the system of cities.

Consider a model where each city is characterized by wages, amenities, and rents that respond to the mix of skilled and unskilled workers living there, and suppose it matches the observed behavior of the economy between 1980 and 2000.

1. Suppose we assign 2000 productivity levels to the 1980 version of the economy, and let workers choose their destinations. How correlated would this distribution of workers across cities be to the observed distribution in 2000? If it is highly correlated, then labor demand drove most of the observed migration. If not, then the other factors are more important: endogenous amenity changes, agglomeration, rents.
2. Suppose we don't allow amenities to respond endogenously to the inflow of skilled workers, and ask workers to choose where to live in this alternate world, where amenities are different. How similar would the distribution of skill be to the observed one? If it is very similar, then endogenous amenities are unlikely to be important in explaining the distribution of skill. If it is very different, then endogenous amenities are important.
3. Suppose we keep rents and amenities fixed at their 1980 levels, but allow wages to increase until 2000 as observed in the real world. By comparing each worker's 1980 utility to their counterfactual 2000 utility, we can know how much their welfare changed. We can further compare utility changes between skilled and unskilled workers. This would tell us how much of the welfare gap is attributable to increases in wages. By repeating the exercise, but allowing rents and amenities to change as observed, we can tell whether rents and amenities make the welfare gap larger or smaller.

6.2.1 Model

We need, as stated earlier, a model of a system of cities where each has a different wage, rent, employment level, and amenities. Wages, rents, and amenities respond to the skill mix, and workers choose where to live. Then the model has four parts:

1. Labor demand, describing how wages depend on local employment, for skilled and unskilled
2. Labor supply, describing where skilled and unskilled workers locate as a function of city characteristics
3. Housing supply, describing how rents depend on local employment
4. Amenity supply, describing how amenities endogenously respond to the skill mix

6.2.2 Labor Demand

Let j index cities, and t index time. $w^H_{jt} = \frac{w^H_{jt}}{P_t}$ and $w^L_{jt} = \frac{w^L_{jt}}{P_t}$ are high and low skill log real wages, respectively. H_{jt} and L_{jt} are high and low skill employment. Labor demand describes wages as a function of employment. Each wage depends on both quantities. This reflects both agglomeration, through which the skill mix affects wages, and that both kinds of labor are substitutes.

$$w^H_{jt} = \gamma_{HH} \ln H_{jt} + \gamma_{HL} \ln L_{jt} + \varepsilon^H_{jt} \tag{6.9}$$

$$w^L_{jt} = \gamma_{LH} \ln H_{jt} + \gamma_{LL} \ln L_{jt} + \varepsilon^L_{jt} \tag{6.10}$$

Target parameters to estimate are the four γ. We need some exogenous variation in labor supply to cities in order to identify them. Given that, estimating this is simply a matter of applying instrumental variable estimation.

6.2.3 Labor Supply and Housing Demand

Each worker, indexed by i, chooses where to live according to wages, rents, and amenities to maximize her utility. Let $r_{jt} = \ln \frac{R_{jt}}{P_t}$ be log real rent, and $\mathbf{A}_{jt}$ the vector of local amenities. s_i is a function that maps amenities to utility, and varies with each worker.

We assume that each worker spends a share ζ of income on housing HD_{ijt}:

$$HD_{ijt} = \frac{\zeta w^{edu}_{jt}}{R_{jt}} \tag{6.11}$$

so that total land demand is

$$HD_{jt} = L_{jt} \zeta \frac{W jt^L}{R_{jt}} + H_{jt} \zeta \frac{W jt^H}{R_{jt}} \tag{6.12}$$

Indirect utility for worker i of skill edu if she chooses to live in j at time t is then:

$$V_{ijt} = \left(w^{edu}_{jt} - \zeta r_{jt} \right) \beta^w + s_i(\mathbf{A}_{jt}) + \varepsilon_{ijt} \tag{6.13}$$

We take amenities to mean all characteristics of j that matter to individuals, other than wages and rents. $\mathbf{A}_{jt}$ is composed of two kinds of amenities. The first, $\mathbf{x}^A_{jt}$, is exogenous to the skill mix in the city. These include, e.g. climate and proximity to the coast or natural parks. The second, a_{jt}, includes local amenities that depend on the skill mix, such as school quality, crime, retail, culture, infrastructure, and the local government.

$$s_i(\mathbf{A}_{jt}) = \beta_i^x \mathbf{x}_{jt}^A + \beta_i^a a_{jt} + \beta_i^{st} \mathbf{x}_{jt}^{st} \tag{6.14}$$

Notice that V_{ijt} only depends on i through the individual error term ε_{ijt}, and i's education level and state of origin. The error term in (6.13) is assumed to have a type 1 Extreme Value distribution.

We can group all terms that depend only on j and t:

$$\delta_{jt} = \beta^w \left(w_{jt}^{edu} - \zeta r_{jt} \right) + \beta_i^x \mathbf{x}_{jt}^A + \beta_i^a a_{jt} \tag{6.15}$$

Then, we can reexpress (6.13) in terms of δ as follows:

$$V_{ijt} = \delta_{jt} + \beta^{st} \mathbf{x}_{jt}^{st} + \varepsilon_{ijt} \tag{6.16}$$

6.2.4 Housing Supply

The relationship between quantity demanded and rent depends on how easy it is to build, which in turn depends on local geography and regulation. Define local construction costs CC_{jt}, and local buildings restrictions x^{geo} and x^{reg}

$$r_{jt} = \alpha_t + \ln(CC_{jt}) + \gamma_j \ln HD_{jt} \tag{6.17}$$

$$\gamma_j = \gamma + \gamma^{\text{geo}} \exp(x^{\text{reg}}) + \exp \gamma^{\text{reg}} (x^{\text{reg}}) \tag{6.18}$$

6.2.5 Amenity Supply

An entry of the amenity vector is endogenous, and depends on the mix of skill mix in the city.

$$a_{jt} = \gamma^a \ln(\frac{H_{jt}}{L_{jt}}) + \varepsilon_{jt}^a \tag{6.19}$$

6.3 Identification

Identification of these five sets of equations depends on finding variation in city characteristics that is exogenous to local conditions. Consider first the following shocks to local high and low skilled labor demand:

$$\Delta B_{jt}^{H} = \sum_{ind} \left(w_{ind,-j,t}^{H} - w_{ind,-j,t-1}^{H} \right) \frac{H_{ind,j,t-1}}{H_{j,t-1}} \tag{6.20}$$

$$\Delta B_{jt}^{H} = \sum_{ind} \left(w_{ind,-j,t}^{L} - w_{ind,-j,t-1}^{L} \right) \frac{L_{ind,j,t-1}}{L_{j,t-1}} \tag{6.21}$$

These are called Bartik shocks, and rely on the idea that if an industry grows a lot at the national level, locations with high concentrations of that industry will receive a large shock to labor demand. These are calculated separately for high and low skilled workers. This variable can serve as a demand shifter and will generate exogenous variation in local wages.

Exogenous variation in local wages generate differential changes in rents depending on the local building restrictions, conditional on the wage change. Since there are separate shocks to high and low skill labor, the skill mix and endogenous amenities also change exogenously to each worker's location decision. Lastly, differential immigration due to exogenous differences in the rent elasticity identify labor demand.

6.4 Counterfactuals

With the estimated model, Diamond performs the counterfactuals we stated at the beginning of the section.

She finds that the counterfactual 1980 world with 2000 worker productivity has skill ratios across cities that are very similar to those in the 2000 data. The correlation she finds is .8. This means that local productivity increases can explain most of the geographical skill sorting.

She also finds that by including 2000 rent in the counterfactual, the correlation decreases to .32. However, with endogenous amenities included, the correlation increases to .86. She concludes that endogenous amenities reinforced the geographic skill concentration.

Finally, she compares welfare increases across groups. By considering both rent and amenity changes between 1980 and 2000, she finds that the welfare gap between high and low skill increased more over that period than the wage gap suggests.

6.5 Are cities too big or too small?

We now address whether the size distribution of cities is optimal. Henderson (1974) shows that the sizes of cities may not be optimal due to the presence of externalities. We illustrate this with a simple example (from Matthew Turner).

Consider a system of two cities indexed by $i = 1,2$. Each city has population N_i, and $N_1 + N_2 = 1$. Output in each city is given by:

$$Y_i = \tilde{A}_i N_i^{\theta},\ \theta \in (0,1). \tag{6.22}$$

Because of agglomeration economies, productivity depends positively on the size of the population:

$$\tilde{A}_i = A_i N_i^{1+\sigma}. \tag{6.23}$$

σ should be between 0.03 and 0.09, from our estimates of agglomeration economies from the agglomeration chapter.

Wages are set equal to marginal productivity:

$$w_i = \theta \tilde{A}_i N_i^{\theta-1} = \theta A_i N_i^{\sigma+\theta}. \tag{6.24}$$

In each city, workers pay a commuting cost that is increasing in population (these are congestion externalities).

$$R_i = R_0 N_i. \tag{6.25}$$

Utility u_i is wages minus the commuting cost. In spatial equilibrium, this is equalized across cities. Total welfare in city i is $W_i = N_i u_i$.

A social planner wants to choose population optimally to maximize welfare. The FOC for this problem would be:

$$\frac{dW_i}{dN_i} = u_i + N_i \frac{du_i}{dN_i} \tag{6.26}$$

Equation 6.26 shows that the planner's problem differs from the individual problem. Individuals do not internalize the agglomeration effects. The planner internalizes the agglomeration effects and would like to allocate more people to the city with the higher externality.

To find the optimal city size in this problem, set the FOC to 0 to yield:

$$N^{\star} = \left[\frac{R}{(\sigma+\theta)A\theta}\right]^{\frac{1}{\sigma+\theta-1}} \tag{6.27}$$

With $\sigma + \theta \in (0,1)$, the size of the city is increasing on the strength of agglomeration externalities.

Consider the size distribution of cities. Spatial equilibrium will set utility to be equal across locations

$$u(N_1) = u(1-N_1), \tag{6.28}$$

while a planner would set the sizes such that the marginal gain in welfare across cities from an increase in population is equal, that is

$$\frac{dW_1}{dN_1} = \frac{dW_2}{dN_2} \tag{6.29}$$

In general, the planner's solution need not be the same. When agglomeration externalities are large in a city, this city is going to be too small in equilibrium compared to its efficient size.